

MANSW
Written Submission

Stage 6 Mathematics Review
June 2006

In 2006, the membership of MANSW consists of Mathematics teachers, teacher educators, TAFE teachers as well as mathematicians. As a group, we are very diverse – in our experiences, in our beliefs and naturally in our views on Stage 6 Mathematics, but however diverse we are as a group, it is clear in the many discussions that MANSW has had with its members that we all share a passion for Mathematics and a desire to see our students achieve the best they can. MANSW has a long history of working hand in hand with the Board of Studies in assisting Mathematics teachers to implement curriculum and assessment changes and we look forward to continuing this important relationship throughout the Stage 6 Mathematics review process.

The information in this submission has been collected from online surveys, e-mails to our office, discussions between members and members of the MANSW Executive and a Preliminary Stage 6 Mathematics Discussion Forum that was held earlier in the year. Some of the issues raised in this submission were also highlighted in our oral submission to the Board of Studies.

In an attempt to minimise the inevitable confusion in this document, the term 'Mathematics' has been used as a general term to describe any study within the mathematical discipline, whilst the term '2 Unit Mathematics' has been used to describe the current Board of Studies 2 Unit calculus course offered in Stage 6.

There are ten areas we would like to discuss in this submission:

1. Mathematics is for everyone
2. Time to learn
3. Communication with the wider educational community
4. The role of technology
5. The structure of the new suite of Mathematics courses
6. Statistics and other topics
7. Course specific comments
8. Consultation and the syllabus development process
9. Syllabus implementation issues
10. Other issues raised

Mathematics is for everyone:

First and foremost, MANSW believes that there should be a Board Developed Stage 6 Mathematics course for ALL students in NSW, regardless of their ability. We must cater for the students who do not intend to complete further mathematical studies at university just as well as we cater for those who go on to university. Currently, students achieving Stage 5.1 outcomes at the end of Year 10 are prepared only for the Preliminary component of the Stage 6 General Mathematics course. These students require further development of their mathematical skills to adequately prepare them for life as citizens and consumers in an information age. They require more time to develop their mathematical skills, learning to apply them in more sophisticated ways and in more complex situations. We believe that this current deficiency must be addressed in the new suite of Mathematics courses. We believe it is difficult to address the mathematical needs of 30 000 different students with one course, such as General Mathematics, and believe an additional course is required.

Students need time to learn Mathematics, rather than just memorise it:

Students need time to learn. The quantity of content, knowledge and skills in each course must be carefully balanced to ensure that students have time to develop familiarity, conceptual understanding and confidence with Mathematics. An overcrowded syllabus tends to result in a lecture style of delivery in many classrooms and this does little to build the essential understanding of abstract concepts.

Students need time to develop a comprehensive, cohesive understanding of the Mathematics they study rather than just a memorised knowledge of mathematical procedures.

One of the most important aims of our Stage 6 courses must also be to facilitate positive, enjoyable teaching and learning experiences for the students and teachers in NSW. Our syllabuses should engage students and allow teachers the essential time to utilise the full range of their knowledge, skills and understanding that represent the craft of teaching.

Any Mathematics syllabus that is overcrowded will almost certainly result in a decline in student understanding. MANSW believes that such a situation would be completely unacceptable.

Communication with students, parents, teachers and the wider community:

Clear guidelines in the new syllabus documents indicating preferred, optimal and/or optional pathways from different Stage 5 pathways into the new Stage 6 courses will be essential for teachers, parents, students and the community to understand the intended relationship between the new Stage 5 and 6 Mathematics courses.

The role of technology:

- Within our membership, there is a wide range of opinions on whether certain handheld technologies, such as Graphics Calculators, Computer Algebra Systems or handheld spreadsheets, should be permitted in the HSC examinations. Some members feel that scientific calculators have sufficient capabilities. Others find it difficult to imagine how any course for the 21st century would not attempt to use the full range of portable technological devices. Most importantly, MANSW feels strongly that whatever the decision about the level of technology that will be permitted during the HSC examinations, it *must* be informed by research and the decision must be made very early in the syllabus development process. It is essential for teachers to know the tools their students will have before they can participate effectively in any consultation or comment on draft syllabuses.
- We believe that it is education professionals in conjunction with educational researchers who are best equipped to decide what level of technology is the pedagogically best practice for students studying Mathematics in NSW schools. The BOS should be able to approach all calculator manufacturers, stating the exact nature of the requirements of the handheld devices we want for our courses.

- Decisions about the role of technology in the HSC examinations must also ensure a level playing field for all students in NSW. We believe that the current situation in General Mathematics, with the optional use of Graphics Calculators, is unfair. Teachers who use the technology and are familiar with all it can do argue that students who do not use Graphics Calculators in this course are at a disadvantage in their learning experiences in the classroom, as well as being disadvantaged in the HSC examination. This inequity must be addressed.
- If a decision is made to allow the use of technologies with greater capabilities than a scientific calculator in the calculus based courses, then the communication of this decision as early as possible is essential for two reasons. Firstly, it will allow teachers the essential time for quality professional learning to ensure they are familiar with the technology and all it can do. Secondly, it will allow all systems the essential lead time to investigate, develop and implement appropriate models that ensure that such technologies are affordably placed in the hands of every students in NSW. An indication of the role of technology in the courses will certainly need to be provided to the syllabus writers very early on in the process and MANSW believes that teachers and systems should be informed at this same early stage. One possibility may be that decisions relating to the use of technology in each course could be made at the same time as the Broad Syllabus directions are approved. This would maximise planning time before the syllabuses are implemented.
- It is important to ask the question 'what mathematics do we want our students to understand and how will we test their understanding?' In the current 2 Unit Mathematics course, there is a strong emphasis on algebraic manipulation. Some MANSW members have argued that this course needs a greater balance between the skills of algebraic manipulation and the application of these skills in modelling situations. The introduction of technology with additional features does not prohibit the testing of these algebraic manipulation skills. For example, these two different components could be addressed in an HSC examination with two sections, one section utilising technology and one section that is completely technology free, as is the current case in the School Certificate Mathematics test.

The structure of the new suite of Mathematics courses:

- Any new structure of courses must provide the essential challenge and rigour for our most capable mathematicians. We would expect multiple extension courses will be maintained.
- Any new structure must facilitate some student movement between courses, particularly during the Preliminary course as there are many students who do not initially select the most appropriate course.
- Any new structure of courses must encourage students to study the discipline of Mathematics at the highest level possible.
- Difficulties with the current 2 Unit Mathematics course have resulted in a serious decline in numbers over the last 5 years.
For example:
 1. The total number of candidates in the 2005 HSC 2 Unit Mathematics course was approximately 30% lower than the number of candidates in 2004 Preliminary 2 Unit course. (See Appendix A)
 2. In the last 10 years, when the total number of students sitting the HSC appears to have risen by about 5000, the total candidature of 2 Unit Mathematics course has dropped by 5000. (See Appendix B)

We believe that these figures indicate significant difficulties with the current 2 Unit Mathematics course. Some of these students have moved to General Mathematics and others have left Mathematics altogether. MANSW does not think this situation is desirable or acceptable.

- In the current 2 Unit Mathematics course, the distinction between Preliminary / HSC course content has been problematic. Since content from both the Preliminary and HSC course can be examined in the external examinations, perhaps schools should have the flexibility to programme the courses in the most appropriate sequence for their students rather than programming all Preliminary then all HSC topics.
- We believe that the relationship between a base 2 Unit course and its corresponding Extension courses needs to be examined carefully. If it is the case that the role of the 2 Unit HSC examination is to spread both the 2 Unit and Extension candidates then the nature of the content used to discriminate between these two groups of candidates must not be biased in favour of the Extension candidates. We believe that any mathematics examination at any stage of schooling, should give the diligent students of average ability an opportunity to show what they know and can do. We do not believe that this is currently the case and that this may also have contributed to the decline in numbers in the 2 Unit Mathematics course.
- In considering changes to the total number of Board Developed Stage 6 Mathematics courses, many teachers indicated that it was unreasonable to consider any more than five Mathematics courses for Stage 6 for a number of reasons. Firstly, teachers indicated that giving students too many options at the start of Year 11 meant that students may cut off options for further study too early. Also many teachers indicated that they would not be able to staff or support more than five courses in their school.
- Many members provided suggestions for a new structure and relationship between the Stage 6 Mathematics courses. The overwhelming majority of members felt that the structure of the Extension 1 and Extension 2 course suited the needs of the students presently enrolled in these courses and consequently should not be altered. Whilst suggestions of multiple Extension 1 courses (such as Statistics, Pure Mathematics, Applied Mathematics) would be possible, many schools feel that they would not be able to support three different Extension courses. Suggestions for the relationships between three different 2 unit Mathematics courses included:
 - An updated 2 Unit Mathematics course, an updated General Mathematics course with a harder Preliminary course and a new 2 Unit course for 5.1 students
 - An updated 2 Unit Mathematics course, a new 'in between 2 Unit and General Mathematics' course and a General Mathematics course with an easier HSC component for 5.1 students
 - An updated 2 Unit Mathematics course, an updated General Mathematics course and a 1 Unit 'Fundamentals of Mathematics' course to be taught concurrently with General Mathematics for the Stage 5.1 students
 - An updated 2 Unit Mathematics course, an updated General Mathematics course and a vocational course with an optional external examination. The core of such an applied mathematics course could include basic numeracy, data and financial literacy and possible options could link directly with other vocational courses such as Hospitality, Business Services, Information Technology or Construction
 - A 2 Unit calculus course and a 2 Unit non-calculus course (possibly including some statistics), with students being able to take either or both of these courses
 - An updated 2 Unit Mathematics course and then two other 2 Unit courses with a structure similar to Advanced / Standard English with some common topics and assessment questions.

Members also suggested that electives or options in any of the Mathematics courses might be another way to ensure that schools have the flexibility to address the different student needs. These options all require further investigation to ensure that a structure exists that provides an appropriate, challenging Stage 6 Mathematics course for all students.

Statistics and other topics:

- The content in the three 2 Unit courses must reflect the differing needs of the students studying them. The issue of whether topics such as Statistics, Logic, Number Theory and Financial Mathematics are under-represented in a number of our current courses requires further investigation. Some members believe that Statistics should appear in all Stage 6 Mathematics courses, whilst others advocate a separate Statistics course.
- In considering whether topics such as Statistics should be included in our courses, it is important to remember that topics can be taught in quite different ways. It may be appropriate for some students to be introduced to the derivations of statistical methods, whilst others only require experience in using statistical methods. This is true of many mathematical topics. The strengths and weaknesses of the statistical components in the International Baccalaureate Mathematics courses may prove useful information in making the decision on the role of Statistics in the NSW courses.

Course specific comments:

The 2 Unit, Extension 1 and Extension 2 courses (previously called 2 Unit, 3 Unit and 4 Unit Mathematics) have existed for many years. In this time, the content and relationship between these courses has remained essentially the same. Any changes to this structure and the implications for student choice must be communicated clearly to the community.

Many MANSW members have provided additional comments which relate to particular issues with the current Stage 6 courses.

Extension 2 course:

- Any new structure of courses must provide the essential challenge and rigour for our most capable mathematicians in Extension 2 Mathematics.
- Some members consider that topics such as Matrices, Vectors and Boolean Algebra should be considered for inclusion in the Extension 2 course. Others believe applications other than dynamics should also be included.
- The possibility of having a two year Extension 2 Mathematics course should be investigated. This would allow students to study four units of Mathematics in both Year 11 and 12, as is the case with other disciplines such as History.

Extension 1 course:

- Any new structure of courses must provide the essential challenge and rigour for our most capable mathematicians. We would expect multiple extension courses will be maintained.
- The current Extension 1 course seems to cater reasonably well for the needs of the students who elect to study this course.
- The possibility of an Extension Mathematics course, worth 3 units, as a stand alone course should be investigated. In such a structure, there would be no common examination with any 2 Unit Mathematics course.
- The possibility of including some study of Complex Numbers in an Extension 1 course could be considered.

2 Unit Mathematics course:

- The name 'Mathematics' for a course causes considerable confusion in schools and the wider community as to whether it refers to a discipline or a course. It should not be used again.
- There are significant differences between the Stage 4/5 courses that existed when the current 2 Unit Mathematics course was written and our current Stage 4/5 Mathematics courses. It is essential to consider the common / realistic Stage 5 exit points when developing our new Stage 6 courses.
- Teachers are reporting that many of the needs of 2 Unit Mathematics students are not currently being met by this course as mentioned above.

- The Geometry taught in the current course does not build significantly on the Geometry addressed in Stage 5.3. Some members believe that it should not appear in Stage 6, whilst others argue that the use of portable handheld dynamic geometry packages enable students to build significantly on the current level of geometry in the course.
- Some members have concerns over aspects of this course (such as the locus of a parabola) which are not essential to a cohesive understanding of calculus. Others suggest that perhaps the applications of calculus to motion should be replaced with other applications of calculus (perhaps such as statistical or financial applications) that may have greater relevance for more students.

General Mathematics course:

- General Mathematics was the only new Stage 6 Mathematics course developed for the New Higher School Certificate that was first awarded in 2001. Having now been in place for a number of years, MANSW would strongly support an external, independent evaluation of this course. We believe that this would provide very useful data for this review process.
- The difference in the degree of difficulty between the Preliminary General Mathematics course and the HSC General Mathematics course is far too great. It gives Year 11 students an unrealistic assessment of the level of success they are likely to experience in Year 12. This situation must be rectified.
- MANSW believes there are many strengths of the current General Mathematics course, but that it cannot address the needs of all the students who currently have no other Stage 6 mathematical option. In deciding on a new structure, we believe it is possible to either:
 - (a) increase the level of difficulty in General Mathematics, particularly in the Preliminary course, and add a new course.
 - (b) decrease the level of difficulty in General Mathematics, particularly in the HSC course, and add a new course 'in between' this course and current 2 Unit Mathematics course or its replacement.
- If a decision is made to increase the difficulty of General Mathematics (with a new course added 'underneath') then additional concepts and/or topics that could be included are the mathematics of gambling, the concept of calculus and rates of change, further statistics and further financial mathematics.
- Some concerns were raised over aspects of the General Mathematics syllabus that conflicted with standard definitions and formulae, particularly in the area of financial mathematics.
- Some MANSW members feel that the name General Mathematics gives parents, students and the community an incorrect indication of the level of difficulty of this course. The name(s) of any new courses need to be considered very carefully.

Consultation and the syllabus development process:

- Extensive consultation with teachers, academic mathematicians and teacher educators is essential at all stages of the syllabus development process.
- We know that a crucial part of the syllabus development process will involve reviewing research into current practice throughout Australia and overseas. This is particularly vital on the issue of the role of technologies such as Graphics Calculators, Computer Algebra Systems and spreadsheets. We strongly support extensive consultation and communication with all teachers once this research phase has been completed.
- The range of abilities of 60 000 students in NSW is extremely broad and their post school destinations are extremely diverse. This means that at all stages of the consultation process it is really important not to confuse which group of students a particular argument or point of view refers to. We are certain that the mathematics and the approaches to teaching mathematics that are appropriate and necessary for our less able mathematicians are clearly not best practice for the most capable, and vice versa.

Syllabus implementation issues:

- If a decision is made to allow the use of any additional technology in any Stage 6 Mathematics HSC examination, early communication of such a decision to all teachers in NSW is essential. We believe this could be done before work is even begun on the draft syllabuses. This allows the essential time (approximately 3 full years) for quality professional learning to occur. It also allows time for all systems to budget and ensure that this technology is placed in the hands of every student in NSW.
- The implementation timeline must allow time for teachers to do the essential planning, programming and professional learning to ensure that the highest standard of mathematics teaching is preserved in NSW. MANSW feels that teachers need *at least* a full school year from the time of release of the syllabuses before implementation with the first Preliminary cohort. The essential support documents, including assessment and examination specifications and Specimen papers, must be available to teachers as soon as possible – certainly well before the first cohort begins the HSC component of the courses.

Other issues raised that are outside the scope of the current review:

- It is our responsibility as educators to ensure that our schools in NSW help to develop Australia's next generation of scientists, doctors and researchers, of innovators, practical problem solvers and people who can develop pioneering technologies for the 21st and 22nd century. In such a society, many MANSW members have questioned the decision to remove the requirement for all students to study at least one science-based subject for the Higher School Certificate. We would strongly support a review of the removal of this breadth of study requirement.
- Many teachers have questioned the value of the School Certificate credential and the value of the School Certificate Mathematics test. Some have suggested that a Year 10 credential should be a school based credential, distributed only to those students who do not continue into Year 11. We would strongly support a review of the need and role of a state-wide Year 10 credential.

The study of Mathematics is an essential part of life. The skills of mathematical problem-solving, mathematical reasoning, developing mathematical strategies as well as computational competence are essential skills in every member of our society. As a discipline, Mathematics is uniquely placed to provide opportunities for the development of reasoning abilities and logical, flexible thought processes in our students, particularly in Stage 6. The way in which the *Sudoku* phenomena, a simple exercise in logical thinking, has travelled the globe and captured the imaginations of many, is a typical illustration of how the study of mathematical reasoning and logical reasoning has the potential to encourage curiosity, exploration, discovery and invention. Our students need to be involved in the essential mathematical processes of exploring, manipulating, discovering, generalising, abstracting, calculating, predicting, describing, deducing and proving – just to name a few! The study of Mathematics involves invention, intuition, creativity and discovery. Mathematics is essential in many diverse fields, from economics to construction to medicine, because it has the ability to simultaneously represent, interpret, explain and predict.

Comparative studies have suggested that there are many aspects of our current Stage 6 Mathematics courses that are held in highest regard, both within Australia and internationally. This current review process is a wonderful opportunity to preserve these strengths and improve in any areas of weakness. MANSW looks forward to the new suite of Stage 6 Mathematics courses that cater for all students in NSW and encourage students to study these rigorous courses at the highest level they are capable of doing. We hope to increase the overall percentages of students who choose to study Mathematics in Stage 6. Most importantly, as classroom teachers, we look forward to further developing our students' confidence and enjoyment of learning mathematics and doing mathematics.

Appendix A: Changes in Preliminary to HSC Candidature

	<i>n</i> estimation*	Life Skills	GM	2U	Ext 1	Ext 2
2005 HSC	61 821	1 038	28 673	19 006	9 365	3242
2004 Preliminary	68 923	1 191	31 971	27 897	13 202	-
	<i>n</i> estimation*	MIP	MIS	2U	3U	4U
1996 HSC	55 718	4 886	22 907	24 672	10 518	2240
1995 Preliminary	60 375	6 024	23 837	29 896	12 948	-

* Estimated based on the sum of candidatures in mutually exclusive English courses.

Source of raw data for number of students in each course:

http://www.boardofstudies.nsw.edu.au/ebos/static/ebos_stats.htm

Appendix B: Candidature over time

	<i>Estimation of Total HSC Candidates*</i>	Life Skills	GM	2U	Ext 1	Ext 2	<i>Estimation of Total Mathematics Candidates</i>
2005	61 821	1 038	28 673	19 006	9 365	3 242	51 959
2004	62 137	1 006	29 376	19 751	9 959	3 514	53 647
2003	63 675	959	30 849	21 038	10 200	3 432	56 278
2002	61 003	885	31 233	20 141	9 153	2 960	55 219
2001	59 068	750	29 375	20 801	8 794	2 578	53 504

	<i>Estimation of Total HSC Candidates*</i>	MIP	MIS	2U	3 Unit	4 Unit	<i>Estimation of Total Mathematics Candidates</i>
2000	59 154	6 051	27 493	22 485	9 476	2 221	58 250
1999	58 660	5 908	26 737	22 935	9 542	2 170	57 750
1998	57 382	5 746	25 064	23 614	9 873	2 108	56 532
1997	55 717	5 092	23 791	23 716	9 920	2 220	54 819
1996	55 718	4 886	22 907	24 672	10 518	2 240	54 705

* Estimated based on the sum of candidatures in mutually exclusive English courses.

Source of raw data for number of students in each course:

http://www.boardofstudies.nsw.edu.au/ebos/static/ebos_stats.htm